Guillermo Gallego

Research Statement

Previous Research Experience

My research lies within the fields of algebraic geometry, differential geometry and mathematical physics. More precisely, during my doctoral studies I have studied several analogues and generalizations of the **Hitchin fibration**. Related topics include gauge theory, the theory of Higgs bundles, spectral and cameral covers, Langlands duality, geometric invariant theory, the theory of stacks and higher Teichmüller theory.

During my Master's thesis and the first year of my PhD, I studied the Hitchin fibration for **Higgs bundles twisted by a vector bundle**, over a curve. These are pairs (E, φ), where E is a principal bundle and φ is a section of the adjoint Lie algebra bundle of E which is twisted by a vector bundle V which can be of arbitrary rank, instead of taking V equal to the canonical line bundle, as in the usual Higgs bundle case. This is an interesting situation because it lies halfway between the theory of usual Higgs bundles over a curve and over a higher-dimensional manifold. Moreover, the corresponding Hitchin equations have also appeared in the high-energy physics literature, turning it into a problem with physical significance. I studied the Hitchin fibration and the spectral correspondence associated to these objects, together with my advisor Oscar García-Prada and with the late professor M.S. Narasimhan. This work lead to a paper which will appear soon in the International Journal of Mathematics.

In my doctoral studies I have explored several questions related with the **multiplicative Hitchin fibration**. This is a natural analogue of the Hitchin fibration over a curve, in which the Higgs field now is a section of the adjoint bundle of groups, rather than of Lie algebras. Some of the reasons why this fibration is interesting are that it is closely related to the theory of **singular monopoles on 3-manifolds**, as studied by Charbonneau-Hurtubise, T. Mochizuki, and others, and its recent application to the proof of the **Fundamental Lemma of Langlands–Shelstad for the groups**, in the thesis of Griffin Wang. **Vinberg's theory of reductive monoids** also plays a central role in the study of the multiplicative Hitchin fibration.

In my Thesis I have considered a generalization of the multiplicative Hitchin fibration to consider Higgs fields taking values on **symmetric varieties**. The corresponding objects pose a "multiplicative analog" of Higgs bundles for **real forms**. In order to do this, I have studied in great detail the theory of embeddings of spherical varieties, in general, and of symmetric varieties in particular, and I have revisited Nicolas Guay's construction of the **enveloping embedding of a symmetric variety**, a generalization of **Vinberg's enveloping monoid** associated to a reductive group. I have also described the action of certain involutions on the moduli space of multiplicative Higgs bundles, also from the point of view of mini-holomorphic bundles and singular monopoles.

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Research Goals

During this academic year, I am completing the post-doctoral stage of my contract at Universidad Complutense de Madrid. My research prospects for this year include several ongoing projects. In collaboration with Benedict Morrissey, we are trying to prove an analogue of Donagi-Pantev's Langlands duality of Hitchin fibrations for the multiplicative Hitchin fibration. I am also exploring a "non-abelian Hodge theory" for multiplicative Higgs bundles in collaboration with Oscar García-Prada and with professor Jacques Hurtubise. Finally, also in collaboration with professor García-Prada and with one of his current PhD students, we are exploring the Hitchin fibration associated to Vinberg θ-pairs.

In future work, I would like to answer some of the questions left open at the end of my thesis, as well as apply the tools and experience I have gained during my PhD to other, maybe unrelated, problems. In particular, I expect that some of the results of my thesis can be extended from symmetric varieties to the more general context of **spherical varieties**, or more generally to other homogeneous spaces, maybe even to the context of G-Hamiltonian spaces, which play an important role in the **relative Langlands program** of Ben-Zvi, Sakellaridis and Venkatesh. In particular, given the deep connections between the (usual) Hitchin fibration and the multiplicative Hitchin fibration with the Langlands program, I expect that there exist analogues of these in the relative situation.

Another interesting problem arising directly from my PhD Thesis is the detailed description of the multiplicative Hitchin fibration for symmetric varieties. The main ingredient towards this description is the study of the **regular quotient** of a symmetric variety G/H by the natural action of H. This problem can be understood as both a multiplicative analogue of the Hitchin fibration for symmetric pairs, studied in the thesis of Thomas Hameister, and as a generalization of the multiplicative Hitchin fibration, as studied by Alexis Bouthier, Jingren Chi and Griffin Wang.